The Parameterized Complexity of the Induced Matching Problem in Planar Graphs

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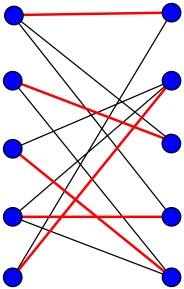
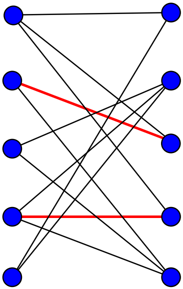
Motivation

Maximum Matching
“marriage problem“



Maximum Induced Matching
“risk-free marriage problem“
[Stockmeyer, Vazirani, IPL 15, 1982]



Maximum Induced Matching

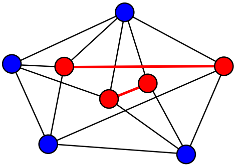
Input

An undirected graph $G = (V, E)$ and a nonnegative integer k.

Question

Is there an edge subset $M \subseteq E$ of size at least k, such that

1. M is a matching, and
2. no two edges of M are connected by an edge of G?



Known Results (1)

NP-completeness on Restricted Graph Classes

- Planar graphs of maximum degree 4

  [Ko, Shepherd, SIAM Journal on Discrete Mathematics 16, 2003]

- Bipartite graphs of maximum degree 3, C_4-free bipartite graphs

  [Lozin, Information Processing Letters 81, 2002]

- Hamiltonian graphs, line-graphs, and r-regular graphs for $r \geq 5$

  [Kobler, Rotics, Algorithmica 37, 2003]

Known Results (2)

Polynomial-Time Solvable

- Trees

  [Fricke, Laskar, Congressum Numerantium 89, 1992]
- Chordal graphs

  [Cameron, Discrete Applied Mathematics 24, 1989]
- Weakly chordal graphs

  [Cameron, Sritharan, Tang, Discrete Mathematics 266, 2003]

Approximation Results

- APX-hard on regular graphs

  [Zito, WG'99]
- APX-hard on bipartite graphs of maximum degree 3, factor-r approximation on r-regular graphs ($r \geq 3$)

  [Duckworth, Manlove, Zito, Journal of Discrete Algorithms 3, 2005]

Fixed-Parameter Tractability

Definition

A problem is *fixed-parameter tractable* with respect to parameter k if it can be solved in $f(k) \cdot \text{poly}(n)$ time.

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Problem Kernel

Parameterized problem L. Instance (I, k).

$$(I, k) \xrightarrow[\text{poly}(n) \text{ time}]{\text{reduction rules}} (I', k')$$

- $(I, k) \in L \leftrightarrow (I', k') \in L$
- $k' \leq k$
- $|I'| \leq g(k)$

Linear Problem Kernel: $g(k) = c \cdot k$ for some constant c.

Fixed-Parameter Tractability Results

Known Result

Maximum Induced Matching parameterized by k is W[1]-hard

[M., Thilikos, ACiD'06]

(W[1]-hard problems are presumably not fixed-parameter tractable)

Our Results

- A linear problem kernel in planar graphs.
- An improved dynamic programming on tree decompositions.

Linear Problem Kernels in Planar Graphs

- DOMINATING SET

  [ALBER, FELLOWS, NIEDERMEIER, Journal of the ACM 51, 2004]
- Improved Kernel for DOMINATING SET, lower bound for kernel size

  [CHEN, FERNAU, KANJ, XIA, STACS'05,

  full version to appear in SIAM Journal on Computing]
- FULL-DEGREE SPANNING TREE

  [GUO, NIEDERMEIER, WERNICKE, IWPEC'06]
- Dominating Set problems in graphs of bounded genus

  [FOMIN, THILIKOS, ICALP'04]
- General kernelization framework

  [GUO, NIEDERMEIER, ICALP'07]

The Data Reduction Rules

R0 Remove isolated vertices.

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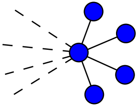
R1 For each vertex, remove all but one degree-one neighbor.



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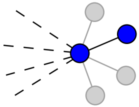

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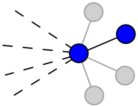
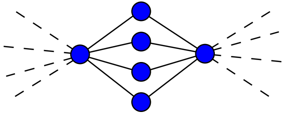


R2 For each vertex pair remove all but one common degree-two neighbor.



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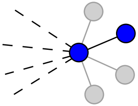

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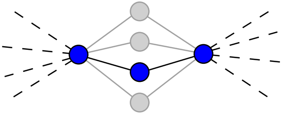
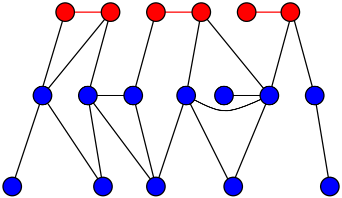
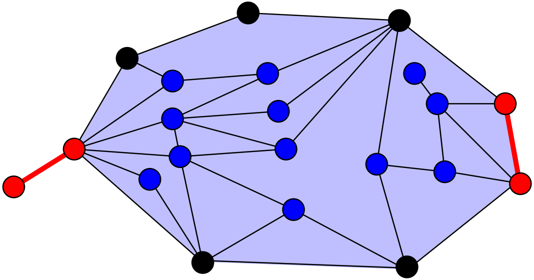


Important Observations

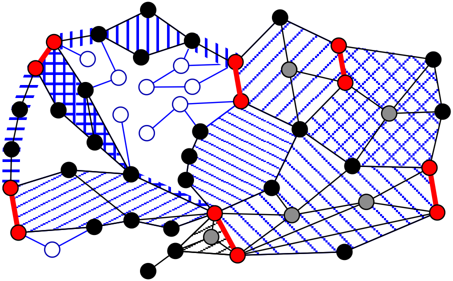
- Every vertex has distance at most two to some vertex in $V(M)$.
- The set of vertices with distance exactly two induces an edgeless graph.



Region



Region Decomposition



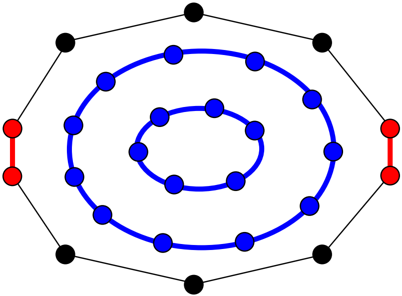
Linear Kernel Proof Framework

Suppose we are given a maximum induced matching M for a reduced graph.

1. Compute a “region decomposition” of the plane graph.
2. Show that there are only $O(|M|)$ regions.
3. Show that in each region there is only a constant number of vertices.
4. Show that there are only $O(|M|)$ vertices outside of regions.

Bounding the Size of a Region

Every vertex has distance at most two to some vertex in $V(M)$
$\Rightarrow$ Outerplanarity argument



Resume

Discussion

- Easy, linear-time data reduction rules.
- Mathematical analysis is quite technical.

Future Work

- Improve the kernel size.
- Search tree algorithm?
- Generalization to non-planar graph classes?

Thank you!